RATIONALIZE WHENEVER NECESSARY

P. 339-340
2-16 EVENS;
22, 24, 26, 28, 32, 37, ~~38~~

2.) $\tan\left(\frac{\pi}{3}\right) = \frac{\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}}$

$= \frac{y}{x} = \frac{\sqrt{3}}{2} \div \frac{1}{2}$

$= \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \boxed{\sqrt{3}}$

4.) $\cot\left(\frac{\pi}{4}\right) = \frac{\cos\frac{\pi}{4}}{\sin\frac{\pi}{4}}$

$= \frac{x}{y} = \frac{\sqrt{2}}{2} \div \frac{\sqrt{2}}{2} = \boxed{1}$

6.) $\csc\left(\frac{5\pi}{4}\right) = \frac{1}{\sin\left(\frac{5\pi}{4}\right)}$

$\frac{1}{1} \div \frac{\sqrt{2}}{2} = 1 \cdot -\frac{2}{\sqrt{2}}$

$-\frac{2}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = -\frac{2\sqrt{2}}{2} = \boxed{-\sqrt{2}}$

8.) $\sec\left(\frac{11\pi}{6}\right) = \frac{1}{\cos\left(\frac{11}{6}\right)}$

$\frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$

10.) $\tan x \cdot \cos x$

$\frac{\sin x}{\cos x} \cdot \frac{\cos x}{1}$

$\boxed{\sin x}$

12.) $3\sin t - 2(1 - 2\sin t)$

$3\sin t - 2 + 4\sin t$

$\boxed{7\sin t - 2}$

14.) $\frac{\sin\theta}{\tan\theta} = \frac{\sin\theta}{1} \div \frac{\sin\theta}{\cos\theta}$

$\frac{\sin\theta}{1} \cdot \frac{\cos\theta}{\sin\theta} = \boxed{\cos\theta}$

16.) $5\cos t - (2\cos t - 3\sin t)$

$5\cos t - 2\cos t + 3\sin t$

$\boxed{3\cos t + 3\sin t}$

22.) IF $\cos\theta = \frac{1}{2}$ $\left(0 \le \theta \le \frac{\pi}{2}\right)$ FIND $\csc\theta$ and $\cot\theta$

$\csc\theta = \frac{1}{\sin\theta}$

Find $\sin\theta$: $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(\frac{1}{2}\right)^2 = 1$

$\sin^2\theta + \frac{1}{4} = 1$

$\sin^2\theta = \frac{3}{4}$

$\sin\theta = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$ ← Positive, since $0 \le \theta \le \frac{\pi}{2}$ (otherwise could be negative)

$\csc\theta = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}\frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$

$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{2} \div \frac{\sqrt{3}}{2} \Rightarrow \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

24.) IF $\sec\theta = 17$ FIND $\sin\theta$ and $\tan\theta$ $0 \le \theta \le \frac{\pi}{2}$

$\cos\theta = \frac{1}{\sec\theta}$ $\cos\theta = \frac{1}{17}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(\frac{1}{17}\right)^2 = 1$

$\sin^2\theta + \frac{1}{289} = 1$

$\sin^2\theta = \frac{288}{289}$

→ $\sin\theta = \sqrt{\frac{288}{289}}$

$= \frac{\sqrt{288}}{\sqrt{289}}$

$\sin\theta = \frac{\sqrt{288}}{17}$ $\pm$ Must be positive

$\boxed{\sin\theta = \frac{\sqrt{288}}{17}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{\sqrt{288}}{17} \div \frac{1}{17} = \frac{\sqrt{288}}{17} \cdot \frac{17}{1} = \boxed{\sqrt{288}}$

26) tangent graph shifted right $\frac{\pi}{4}$

$$\boxed{f(\theta) = \tan\left(\theta - \frac{\pi}{4}\right)}$$

28) a.) $\cos\alpha = -\frac{\sqrt{3}}{5}$; α in Q3

Find $\sin\alpha$ and $\tan\alpha$

($\sin\alpha$ must be negative)

$$\left(-\frac{\sqrt{3}}{5}\right)^2 + \sin^2\alpha = 1$$

$$\frac{3}{25} + \sin^2\alpha = 1$$

$$\sin^2\alpha = \frac{22}{25}$$

$$\sin\alpha = \pm\frac{\sqrt{22}}{\sqrt{25}} = \boxed{-\frac{\sqrt{22}}{5}}$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = -\frac{\sqrt{22}}{5} \div -\frac{\sqrt{3}}{5}$$

$$= -\frac{\sqrt{22}}{5} \cdot -\frac{5}{\sqrt{3}} = \frac{\sqrt{22}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{\sqrt{66}}{3}}$$

b.) $\tan\beta = \frac{4}{3}$ β in Q3

Find $\sin\beta$ and $\cos\beta$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\left(\frac{4}{3}\right)^2 + 1 = \sec^2\theta$$

$$\frac{16}{9} + 1 = \sec^2\theta$$

$$\frac{16}{9} + \frac{9}{9} = \sec^2\theta$$

$$\frac{25}{9} = \sec^2\theta$$

$$\pm\frac{5}{3} = \sec\theta$$

$$-\frac{5}{3} = \sec\theta$$ → $\cos\theta$ is negative in Q3

$$\boxed{\cos\beta = -\frac{3}{5}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\frac{4}{3} = \frac{\sin\theta}{-\frac{3}{5}}$$

$$\boxed{\sin\beta = -\frac{4}{5}}$$

32) $\cos\theta = \frac{4}{x}$ Find $\sin\theta$ and $\tan\theta$ Q1

$$\left(\frac{4}{x}\right)^2 + \sin^2\theta = 1$$

$$\frac{16}{x^2} + \sin^2\theta = 1$$

$$\sin^2\theta = 1 - \frac{16}{x^2}$$

$$\sin^2\theta = \frac{x^2}{x^2} - \frac{16}{x^2}$$

$$\sin^2\theta = \frac{x^2-16}{x^2}$$

$$\boxed{\sin\theta = \frac{\sqrt{x^2-16}}{x}}$$

$$\tan\theta = \frac{\sqrt{x^2-16}}{x} \div \frac{4}{x}$$

$$= \frac{\sqrt{x^2-16}}{x} \cdot \frac{x}{4}$$

$$\boxed{\tan\theta = \frac{\sqrt{x^2-16}}{4}}$$

37.) a.) $y = \tan t$ has vertical asymptotes at odd multiples of $\frac{\pi}{2}$ ($\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$). This is whenever $\cos t = 0$ (t-intercepts)

b.) $y = \tan t$ has zeros at multiples of π. This is whenever $\sin t = 0$ (t-intercepts)